**Chapter 7 – Liquids**

Gases are quite a bit different from liquids. Gases move erratically, with high velocities, low attractive forces, and do not have a noticeable methodic form of compressing. However with liquids, they represent a sort of middle ground between solids and gases. They take the shape of their container, yet they have structural patterns, and they do display forces of attraction, especially with polar molecules. Liquids in the most general case do share many of the attributes of gases and as such many of the equations are related.

As shown previously when you solve for Vm you get three real roots in the Van de Waals equation of state, and the lowest of the three is the Vm for the liquid state.

As with real gases, **liquids need a compressibility factor** in order to be calculated correctly.

However, this can sometimes be difficult as the Tr and Pr can go outside that of the generalized compressibility charts.

**HEATING LIQUIDS:**

When heating a liquid **two things can occur:**

If the Temperature is **less** **than** that of the **boiling** **point** (**Tb**) the heat transferred will result in a temperature increase. This is known as **Sensible** **Heat.**

If the Temperature is **equal** **to** the **Boiling** **Temperature** (Tb) then the heat transferred will be used to **convert** **the** **liquid** **to** **vapour** and no change in temperature will occur. This is known as **Latent** **Heat**.

Just as with gases **Heat Capacity** is the amount of heat (energy) required to **raise the temperature of 1 mol ( or kg) of the substance by 1 Kelvin**.

For most cases, the **Specific** **Heat** **Capacity** **for** **liquids** **is** **higher** **than** **that** **for** **gases**, the reason that this is like this is because there are more types of energies; this causes the energy to be divided up more and thus less goes to kinetic energy.

**Latent Heat of Vaporization** is the amount of heat required to vaporize one mole of a liquid, and it is denoted as

In order to calculate the total amount of heat needed to convert an amount of liquid from T0 to T1 is to:

Add up the total Sensible Heat to the Boiling Point and after from the beginning temperature and ending temperature, and also to add in the heat required to convert the liquid to vapour.

In order to calculate the Latent Heat of Vaporization () you can use a few different methods:

The simplest is **Trouton’s** **Rule**:

This is not perfectly accurate, but for the majority of liquids it is accurate to within 5%.

Another equation that can be used is the **Clausius**-**Clapeyron** **Equation**:

This is used when we know the Temperature, and the specific Volumes of Vapour and Liquid, and the change in P in respect to Temperature. This method is more widely used. It can also be used to find the **Latent** **heat** **for** **other** **changes** **such** **as** **sublimation** **and** **fusion**.

There is a problem with the equation though:

The Specific Volumes can be hard to calculate, thus we can make some assumptions:

-The Specific Volume of vapour is much larger than liquid

-When at low pressure the specific volume can be approximated

This leads to the simplified equation:

This means that if we know two different sets of data for (P1, T­1) and (P2, T2) and we assume that for a **small** **temperature** **range** **is** **constant** then we can rewrite the equation as:

Then subtracting the two first from the second gives us:

* **If however, the latent heat of vaporization is not constant,** then we can state it as a function of temperature:

Which when substituted into the equation and integrated yields:

* **If the second assumption made about the Vg is not true** then we have to use a completely different equation:

**LIQUID MIXTURES:**

In mixtures of 1 or more fluids, different equations of state must be used, as the composition is no longer a single type of molecule. There are two laws that deal with the partial pressures of the mixture in relation to their mole fraction. These are known as **Raoult’s Law** and **Henry’s Law.**

**Raoult’s Law:** This is used when xi is close to 1

This can be equated to partial vapour pressure:

And since then:

Is known as **relative volatility** and it is the ratio of two substances in respect to their likely hood to evaporate.

**Henry’s Law:** This is used when xi is close to zero

This is the relationship between Henry’s Constant and mol %

This is used when there is a binary mixture with solute(1) and solvent(2).

**Chapter 8 – Motion of Fluids**

As is stated by Newton’s Law, in order for an acceleration to occur there must be an applied force. The same applies for fluids. A fluid is said to be **stressed** when a force is applied to it. There are two kinds of stresses:

**Normal** **Stress** – This is stress that is applied to a fluid perpendicular to the surface of contact, and is equivalent in magnitude to the force applied.

**Shear** **Stress** – This is stress that is applied to a fluid unevenly and not necessarily perpendicularly. This kind of stress causes a deformation and change in velocity depending on the location of the fluid to the point of stress. Shear Stress is denoted by the letter

Since shear stress causes a **change** **in** **velocity** (velocity gradient) depending the perpendicular distance from the “plate” it can be said that shear stress is a function of the **velocity** **gradient** (otherwise known as the **shear** **rate)** (**du/dy**) and can be expressed as:

Which if placed into a linear model would take the form:

Where C is a constant of proportionality that is different for each fluid.

An **ideal fluid** is also known as an **inviscid fluid** because it is said to have no shear stress dependent on the velocity gradient.

**Newtonian Fluids:**

A fluid is said to be **Newtonian** if it is **linearly dependent between the shear stress and shear rate.** As well the **viscosity** is said to be **constant.**

**Non-Newtonian Fluids:**

A fluid is said to be **Non-Newtonian** if the **viscosity** **is** **not** **constant**. This means that depending on the shear stress and shear rate the viscosity could be very different. This makes it impossible to give an equation for viscosity; instead there is an equation for **apparent viscosity.** This is the viscosity at an instant and is said to be:

The reason that the viscosity is not constant is because fluids can behave in many different ways:

**Thixotropic Fluids –** These are fluids that after a certain amount of time undergoing a constant shear stress have a decrease in apparent viscosity due to an increase in shear rate.

**“Thixotropic Fall”**

**Rheopectic Fluids –** These are fluids that after a certain amount of time undergoing a constant shear stress show an increase in the apparent viscosity due to a decrease in shear rate.

**“Rheopectic Raise”**

**Viscoelastic Fluids –** These are fluids that partially return to their original form after a shear stress has been removed.

**Power Law Fluids –** These are fluids that approximately follow the power law, in that their shear stress is a power function of the shear rate. They are approximated by the equation:

In this equation K and n are constants that are different for each fluid. In order to solve for these constants a plot of and is made where the y-intercept is and the slope of the line is .

**Dilatant Fluids –** Fluids that show an increase in apparent viscosity with an increase in shear rate. (

**Pseudoplastic Fluids –** Fluids that show a decrease in apparent viscosity with an increase in shear rate. (

**Bingham Plastic Fluids –** These fluids are fluids that act like solids until a certain amount of stress is applied to the substance at which point they begin to flow. This can be expressed as:

Only when otherwise  **if**

**Potential Flow:**

In order to describe the movement of real fluids an equation first has to be developed for an ideal or inviscid fluid. This is known as the potential flow, and it is when there is no shear stress.

**Bernoulli’s Equation:**

Bernoulli developed an equation to describe the potential flow of a fluid in a tube. He stated that when a fluid is flowing it must have energy. That energy has to be distributed into **kinetic** **energy** (movement), **potential** **energy** (height displacement) and **work** (pressure being applied to fluid). Since the fluid is ideal, there is no friction and as such the amount of energy entering the tube must also leave it. If the entrance of the tube is point **A** and the end of the tube is point **B** then:

**Work** – In order for the fluid to flow, work must be done. This work is caused by the pressure exerting force that causes a volume of liquid to move.

**Kinetic Energy –** The kinetic energy is the energy required to keep the fluid moving.

Since mass is the product of density and volume then:

**Potential Energy –** The potential energy is the energy stored up. It is the product of mass, acceleration due to gravity, and the height.

The equations for the endpoint B are the same form as for point A and when placed together can be said as:

But because it is an ideal fluid it can be said that **mass** **is** **conserved**:

Which means that the equation, when simplified, is:

This is **Bernoulli’s** **Equation**.

**Flow of Real Fluid (Viscous Fluids):**

Unfortunately in real world situations, Bernoulli’s Equation doesn’t work as it doesn’t satisfy the presence of the friction and viscosity. When speaking of real fluids flowing through pipes the flow is said to be **laminar** or **turbulent**.

**Laminar Flow –** This occurs are low velocities when the fluid flows in parallel straight lines.

**Turbulent Flow –** This occurs at high velocities when the fluid moves erratically in the pipe due to the formation of eddies.

**Reynolds** was the first to demonstrate the two types of flow. He made an equation that calculates whether the flow is laminar or turbulent. This is called the **Reynolds** **Number**.

- Flow is **Laminar**

- Flow is in the Transition Phase

- Flow is **Turbulent**

Laminar and Turbulent flow situation require different equations to explain their actions.

Since Newton’s Law states that forces must be balanced, then when fluid flows through a pipe the force being applied to it from the back must equal it from the front. This is shown via the equation:

The Shear Stress is largest at the edge of the pipe where the radius is the largest, and the shear stress is zero at the centre of the pipe.

- Is the Shear Stress at the pipe’s wall.

– Is the Radius of the pipe.

Potential Energy must also be included into the equation:

**Laminar Flow:**

It can be shown that for laminar flow the **velocity** can be expressed as:

And the **average velocity** can be described as:

Laminar flow is described using the **Hagen-Poiseuille Equation**:

This can also be described using the Total Flow Rate:

Where average velocity is described:

**Turbulent Flow:**

It can be shown that for Turbulent Flow in Pipes the velocity can be expressed:

Turbulent flow is described by the equation:

In the above equation there is a new variable called the **friction factor ()**

Which when simplified for laminar is:

This means that **Reynolds number and the friction factor are linearly related for laminar flow**. If the Reynolds number is known then the friction factor can be found and vice-versa.

If the Reynolds number is known for a fluid in turbulent flow a **friction** **factor** **chart** can be used to figure out the friction factor, or vice-versa. On the chart there are **three** **different** **lines**. One for **laminar** **flow**, and two for turbulent; these are for **smooth** **pipes**, and **rough** **pipes**.

**Calculating Average Velocity:**

* **Guess a value for**
* **Calculate**
* **Find Friction Factor**
* **Calculate**
* **If is correct you guessed correctly (5% Error). Otherwise Redo.**

**Calculating Diameter:**

* **Guess a value for**
* **Calculate**
* **Find Friction Factor**
* **Calculate**
* **If D is correct you guessed correctly (5% Error). Otherwise Redo.**

**\*\*For the above (and for Q) you can also guess Friction Factor:**

* **Guess a value for**
* **Calculate**
* **Calculate**
* **If Laminar, STOP, and use Laminar Equation**
* **If Turbulent Check to see if is the same, if not try again with that value otherwise you guess correct**

**Power Consumption:**

Power is the amount of energy per time interval needed to pump a liquid a given flow rate over a given length.

The pressure drop is the drop from the **beginning of the PUMP to the end of the PUMP**.

**Chapter 3 – Notions about the nature of Matter**

**Potential Energy:** Potential energy is energy that is stored from work being done against an opposing force. This energy is stored until the work stops being done; this potential energy is then converted to kinetic energy. Potential energy is usually denoted by the Greek lowercase letter *phi*

A good example of potential energy is the energy that is stored up as **two** **magnets** are placed with the same poles facing each other. The magnets will try to oppose each other creating a force and as the magnets are placed closer work is done against the force. When the magnets are released they fly back due to the potential energy that was stored up.

It can be said that potential energy is a function of distance between magnets (**radius**).

It can also be said that potential energy is stored up when you try to hold the two magnets as they attract each other because once you let go the energy is converted to kinetic energy and they fly towards each other.

The change of potential energy is equal to the force multiplied by the change in the distance. It can also then be said that potential energy is the integral of Force over the range of infinity to r:

**Rigid Sphere Model:**

Instead of looking at magnets we are now going to focus on molecules and how their potential energy changes with respect to distance. The most simplistic model that looks at this is the rigid sphere model. It states that molecules are rigid spheres that do not interact with each other. Since they are rigid and do not react it would mean that when they aren’t touching their potential energy is 0, and when they touch they cannot be brought any closer which means that potential energy would be infinite if that was the case. So it can be summed up as:

**Lennard-Jones Model:**

Because it is believed that molecules exert forces both attractive and repulsive forces the rigid sphere model does not accurately represent real world interactions. Because of this a better model was required to explain what was happening. One that tries to do this is the **Lennard-Jones Model.**

**This is a specific Function but there are other equations.**

This model assumes that at any given time a molecule that is non-polar exerts attractive and repulsive forces simultaneously. However, the dominant force is dependent on the distance separating the molecules. The farther away the molecules are from each other the larger the attractive force and the closer you bring them the larger the repulsive force. Through experimentation this has been equated to:

Where:

The positive term on the right is the repulsive force because at the distance gets smaller more work is required and as such the potential energy is larger. The negative term on the right is the attractive force because as the distance gets larger the attractive force gets larger than the repulsive force. This means that the whole right side represents the net work at a given distance. This means that the net force can be equated to:

When you take the derivate you get:

And

Since it is a relationship between forces at some point the net force will equal 0.

With this relationship it is now possible to calculate the minimum potential energy value:

The values for σ and ε can be found in tables for different substances. However, they are represented in the form of ε/k (where k is Boltzmann constant) and:

**Structures of Solids:**

When dealing with solid structures there are three classifications:

**Crystalline** **Solids** – These are solids that have high degree of organization. (salts, silicon, sulfur)

**Polycrystalline** **Solids** – These are solids that have tons of tiny crystals, but macroscopically they do not appear to be crystalline. (metals, polymers)

**Amorphous** **Solids** – These are solids that have no discernible structure but over time become more organized. (Glass)

**Bravais Lattices**

In crystalline solids the molecules arrange themselves in such a way that they continue indefinitely in all three dimensions in a constant arrangement. There are only 14 arrangements that are possible infinitely and these are known as bravais lattices.

These can then be subdivided into categories based on their sides and angles:

Cubic, Tetragonal, Monoclinic, Orthorhombic, Triclinic, Rhombohedral, Hexagonal.

We are only going to focus on cubic arrangements:

The three arrangements are:

**Simple** **Cubic** – This the most basic and has a molecule are each vertex of the cube.

**Body** **Centered** **Cubic** – In addition to a molecule at each vertex there is also a molecule that is located in the center of the cube.

**Face** **Centered** **Cubic** – This has molecules at vertices and has a molecule on each side that has protrudes into the cube which shows the faces of the cross-sections.

**Simple Cubic:**

The atoms of the structure are positioned so that each atom is directly above and beside another atom. The unit cell is positioned so that the center of an atom is located at each vertex.

Based on the fact that there are 8 vertices and each vertex has 1/8th of an atom, then in one **simple cubic cell there is 1 atom**. It can also be seen that the radius of the atoms where a is the length of the side of the unit cell.

**Body Centered Cubic (BCC):**

This is a cubic structure where a molecule is located in the center of the cell and there are molecules located at each vertex.

Since there are 8 vertices that each have 1/8th of an atom and there is an atom in the center then there are **2 atoms in one unit cell**. The distance between two opposing vertices is equal to 2σ. This then can be used to say that if a is the side then

**Face Centered Cubic (FCC):**

This is an arrangement where atoms are located on each vertex and on each face of the unit cell.

Since we have 8 vertices with 1/8th of an atom each, and there are 6 faces with ½ of an atom each the **total number of atoms in the cell is 4**. The distance across one of the faces is equal to 2σ. It can then be said that **.**

**Density and Volumes of Cells:**

The density can be calculated using the formula:

Where **n** is the number of atoms in a unit cell.

**Void Space:**

Since in each of these arrangements there is a portion of empty space it is referred to as void space. The fraction of a unit cell that is void space can be calculated by the formula:

**Coordination Number:**

This is the maximum number of neighbouring atoms that an atom can have in a particular arrangement.

**Simple – 6 BCC – 8 FCC – 12**

**Chapter 9 – The structure and transport properties of solids**

When looking at solids it can be broken up into three distinct classes:

**Crystalline**, **Amorphous**, and **Elastomers**.

**Thermal Expansion:**

When a solid is heated up, the increase in energy causes the molecules of the solid to vibrate more, and this increase in vibration tends to cause a larger separation between molecules. This is known as thermal expansion. This expansion can be approximated by:

The coefficient of expansion is different for each type of metal. This is why thermostats have two different metals bonded since it causes the strip to bend when heated. This is also why there are certain problems involving solids of different types when heated.

**Thermal Conductivity:**

**Thermal conduction is when heat is transferred through neighbouring molecules**. One dimensional heat transfer can be equated using **Fourier’s** **Law**:

**Composite Walls:**

For composite walls (more than one layer) Fourier’s Law can be said to be:

As can be seen above, the amount of energy is the same as Fourier’s Law just that it now encompasses the total resistance of the layers based on their specific width’s and heat conductivity’s.

It can also be written with the Form in which case this is referred to as the **heat flux.**

**Cylinders (pipes):**

When dealing with pipes it is important to know how much heat loss will be lost through pipes.

The idea is the same except that the surface area is circular and is:

So Fourier’s Law for pipes is:

As can be seen above, the distance of heat transport has been replaced by because it can be imagined that heat is emanating from the center of the pipe outwards in all directions thus it has to move a distance of **.**

However, since the heat loss is done through the pipe, the distance is actually the radius of the pipe on the outside minus the inner radius. So it can be said that the heat loss is the integral below:

This idea can be applied to composite pipes as well: